On stochastic tree distances and their training via expectation-maximisation

Martin Emms

April 2, 2012

Simple edit distance

Consider transforming a sequence S into T, $S \Rightarrow T$

At any given moment an initial portion of S has been transformed into an initial portion of T, $S[0..(i-1)] \Rightarrow T[0..(j-1)]$.

Suppose the process is allowed to continue in one of 4 ways

- **delete**

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- **swap** the next symbol of S for the next ungenerated symbol T, if these are different; denote this operation with $(S[i], T[j])$, where $S[i]$ is the next symbol of S, and $T[j]$ is the next symbol of T
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- **match** just skip past the next symbol of S as it is the same as the next ungenerated symbol of T; denote this also with $(S[i], T[j])$

Call the sequence of ops edit-script between S and T.

Scripts and Mappings

sold to *elder*

(s, λ)
(o, e)
(l, l)
(d, d)
(λ, e)
(λ, r)

Scripts and Mappings

sold to *elder*

(s, λ)	s
(o, e)	o
(l, l)	l
(d, d)	d
(λ, e)	
(λ, r)	

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(d, d)	d	d	d	d	d	d	d
(λ, e)						e	e
(λ, r)							r

Scripts and Mappings

sold to *elder*

$$
\begin{array}{lccccccccl}
(s,\lambda) & s & & & & & & & & (\lambda,e) \\
(o,e) & o & o & e & e & e & e & e & & (\lambda,l) \\
(l,l) & l & l & l & l & l & l & l & & (s,d) \\
(d,d) & d & d & d & d & d & d & d & & (o,e) \\
(\lambda,e) & & & & & & e & e & & (l,\lambda) \\
(\lambda,r) & & & & & & & r & & (d,r)
\end{array}
$$

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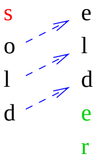

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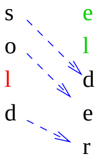

each script corresponds to an order preserving, partial mapping, and vice-versa

Costs for scripts or mappings

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\begin{array}{l}
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$$

s	e
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	r

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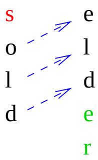

a cost table defines label-dependent costs

for example with table

	λ	a	$\cdots$	z
λ		1	$\cdots$	1
a	1	0	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
z	1	1	$\cdots$	0

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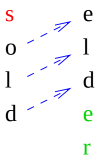

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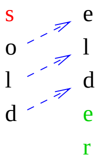

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Definition

(*Sequence-distance*) between $\mathcal{S}$ and $\mathcal{T}$ is the cost of **the least-costly mapping/scirpt** from $\mathcal{S}$ to $\mathcal{T}$

Tree edits

a trees S can be transformed into a tree T, by delete, insert, swap/match operations

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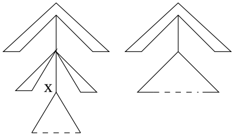

dtrs of x made dtrs of x's parent m

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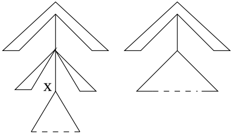

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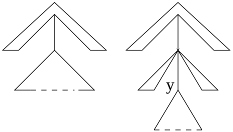

some dtrs of m made dtrs new daughter y of m

swap/match

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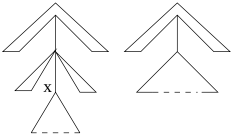

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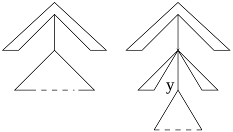

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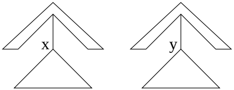
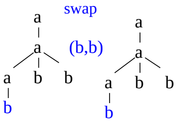
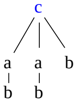


node x turned to node y

Example

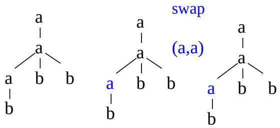


S



T

Example



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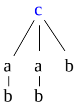
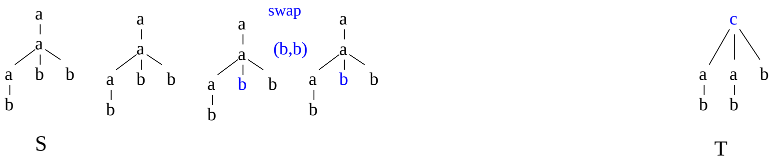


T

Example



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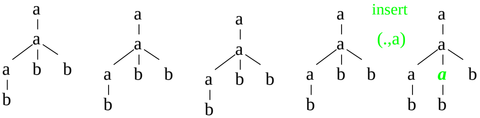

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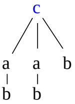

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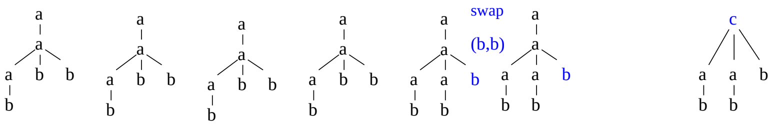

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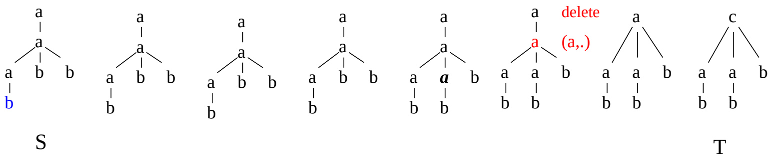

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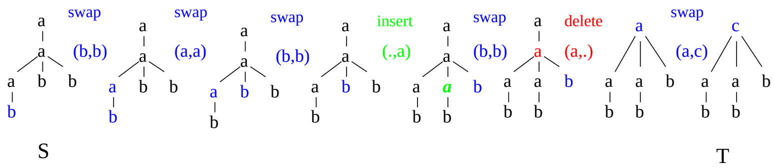

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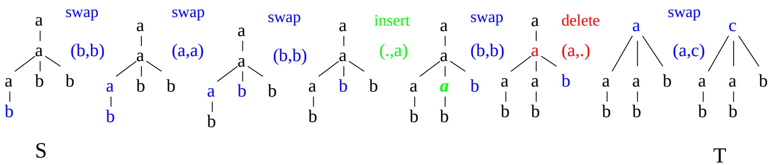
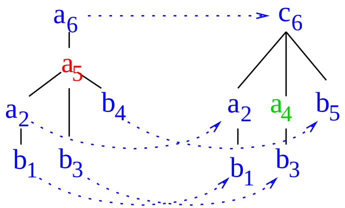


The script encodes a partial mapping $\sigma : \mathcal{S} \mapsto \mathcal{T}$



it is a mapping which respects left-to-right order and ancestry – call such mappings Tai mappings

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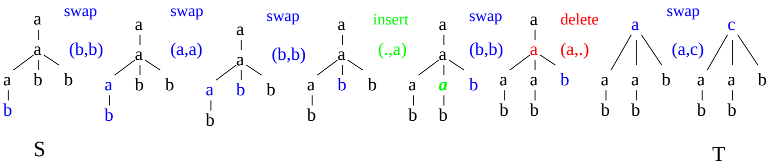

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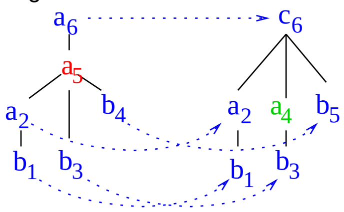

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costs can be assigned to scripts or mappings

Definition

(*Tree- or Tai-distance*) between $\mathcal{S}$ and $\mathcal{T}$ is the cost of **the least-costly Tai mapping (or script)** from $\mathcal{S}$ to $\mathcal{T}$

Stochastic string distances

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$$EdOp = ((\Sigma \cup \{\lambda\}) \times (\Sigma \cup \{\lambda\})) \setminus \langle \lambda, \lambda \rangle$$

  and represent a script with $op_1 \ldots op_n \#$, with each $op_i \in EdOp$.
- assuming a prob distribution p on $EdOp \cup \{\#\}$, define a script probability as

$$P(e_1 \ldots e_n) = \prod_i p(e_i)$$

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- Can think of a script as *yielding* a pair of strings (s, t). If $E(s, t)$ is all scripts which yield (s, t), they defined

  all-paths stochastic edit distance:
  the sum of the probabilities of all scripts $e \in E(s, t)$

  viterbi stochastic edit distance:
  prob. of the most probable $e \in E(s, t)$

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The all-scripts stochastic Tai similarity, $\Theta^A_s(S,T)$, is the sum of the probabilities of all edit-scripts which represent a *Tai*-mapping from S to T. The all-scripts stochastic Tai distance, $\Delta^A_s(S,T)$, is its negated logarithm, ie.

$$2^{-\Delta^A_s(S,T)} = \Theta^A_s(S,T)$$

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  cat(S) = *VOTE*($\{$*categories of k* **nearest neighbours** *of S* $\}$)

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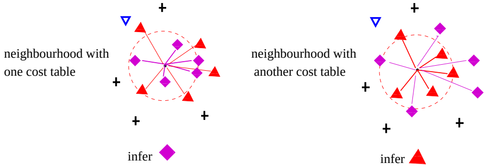

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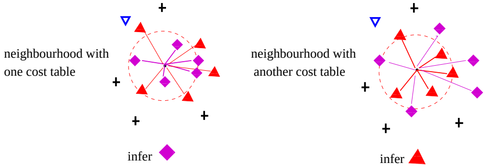

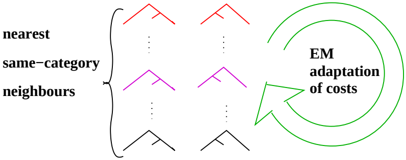

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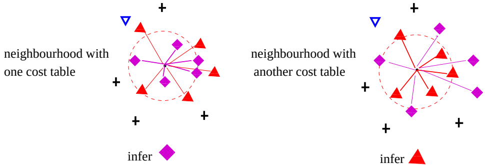

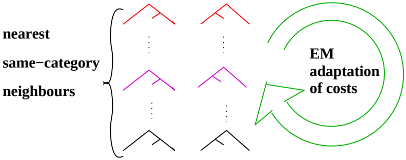

scripts between between *same-category neighbours should have distinctive probs* ⇒ perhaps can use *Expectation-Maximisation techniques to adapt edit-probs from a corpus of same-category nearest neighbours*

Outline

Brute force All-paths EM (infeasible)

Given a corpus of trainings pairs $\mathcal{TP} = \ldots(S, T)\ldots$, let the *brute-force all-scripts EM algorithm*, EM^{A}_{bf}, be iterations of pair of steps

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(Exp)$_A$ *generate a virtual corpus of scripts by treating each training pair* (S, T) *as standing for* all *the edit-scripts* σ*, which can relate* S *to* T*, weighting each by its conditional probability* $P(\sigma/\Theta^A_s(S, T)$*, under current probalities* C^Θ

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A virtual count or expectation $\gamma_{S,T}(op)$ contributed by S, T for an operaton *op* can be defined by

$$\gamma_{S,T}(op) = \sum_{\sigma : S \mapsto T} [\frac{P(\sigma)}{\Theta^A_s(S, T)} \times freq(op \in \sigma)]$$

Brute force All-paths EM (infeasible)

Given a corpus of trainings pairs $\mathcal{TP} = \ldots(S,T)\ldots$, let the *brute-force all-scripts EM algorithm*, EM^{A}_{bf}, be iterations of pair of steps

(Exp)$_A$ *generate a virtual corpus of scripts by treating each training pair* (S,T) *as standing for* all *the edit-scripts* σ*, which can relate* S *to* T*, weighting each by its conditional probability* $P(\sigma/\Theta^{A}_{s}(S,T)$*, under current probalities* C^{Θ}

(Max) *apply maximum likelihood estimation to the virtual corpus to derive a new probability table.*

A virtual count or expectation $\gamma_{S,T}(op)$ contributed by S,T for an operaton *op* can be defined by

$$\gamma_{S,T}(op) = \sum_{\sigma:S\mapsto T}[\frac{P(\sigma)}{\Theta^{A}_{s}(S,T)} \times freq(op \in \sigma)]$$

(Exp)$_A$ accumulates the $\gamma_{S,T}(op)$ for all *op*'s, for all (S,T)

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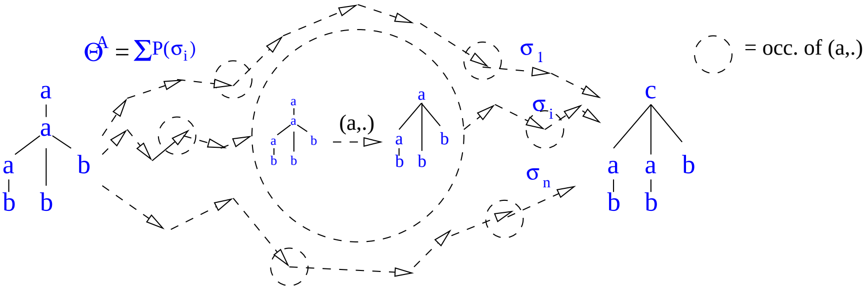

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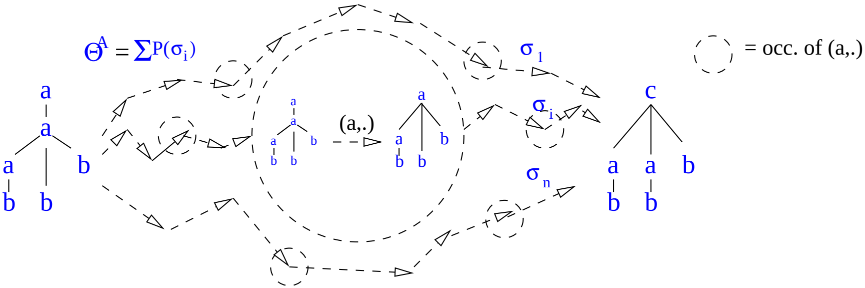

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- **infeasible**

try to split exp. $\gamma_{(S,T)}(op)$ into position specific versions $\gamma_{(S,T)}[i,j](op)$ and then sum

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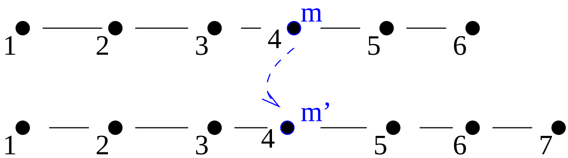

Define $(m_i, m'_j) \in \sigma$, occurrence of posn-specific subst (m, m') in σ as

$$(m_i, m'_j) \in \sigma \text{ if } \sigma = pre \circ (m, m') \circ suff \quad \begin{cases} \text{some } pre \in E(S_{1:i-1}, T_{1:j-1}) \\ \text{some } suff \in E(S_{i+1:I}, T_{j+1:J}) \end{cases}$$

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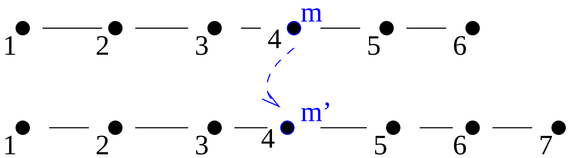

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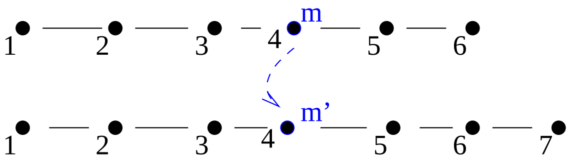

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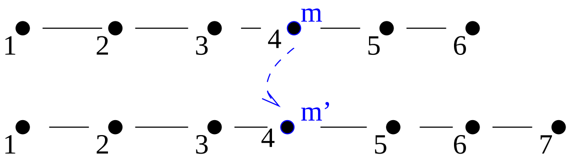

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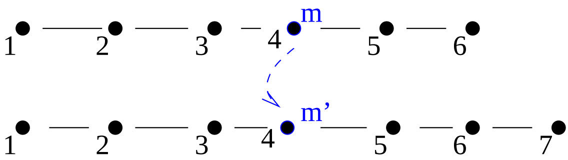

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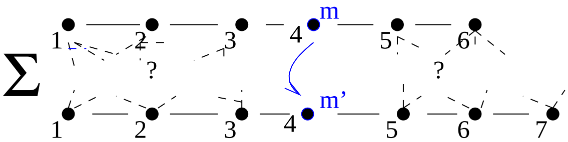

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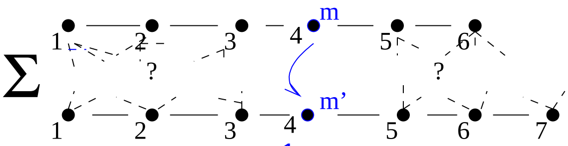

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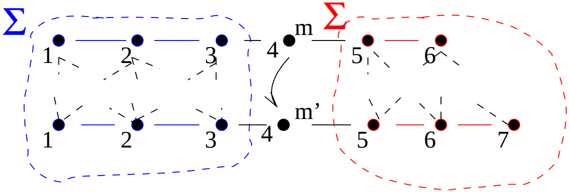

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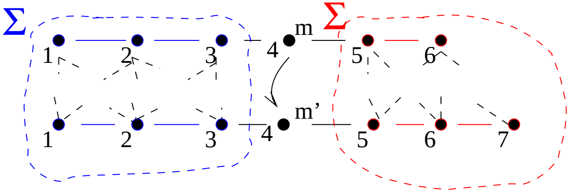

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All-paths EM for linear trees

procedure for determining expectations $\gamma_{S,T}(m, m')$ is then:

- compute table of 'forward' probs: $\alpha[i][j] = \sum_{pre \in E(S_{1:1-1}, T_{1:j-1})} [p(pre)]$
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this has seen widely used to train a string distance measure (ie. linear trees) from a corpus of pairs

Position dept exp. for trees

lets try to apply similar reasoning to stochastic tree distance

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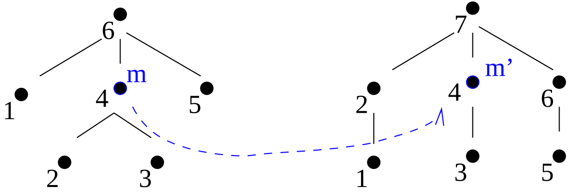

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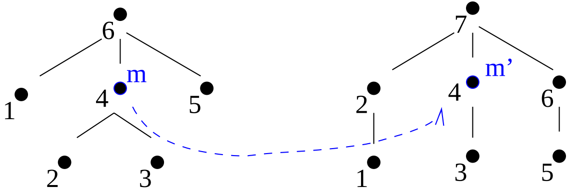

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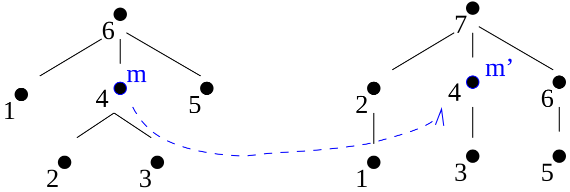

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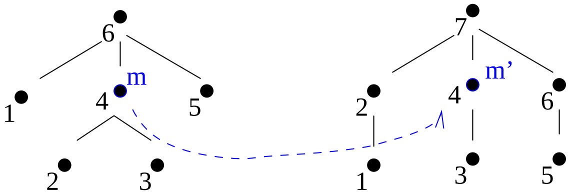

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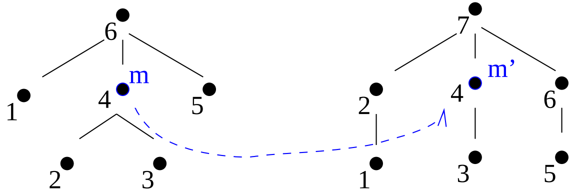

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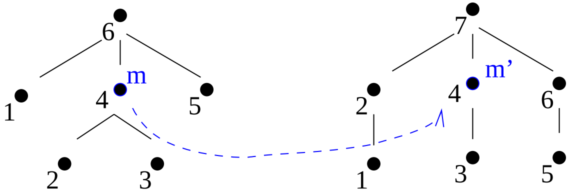

So how to efficiently calculate:

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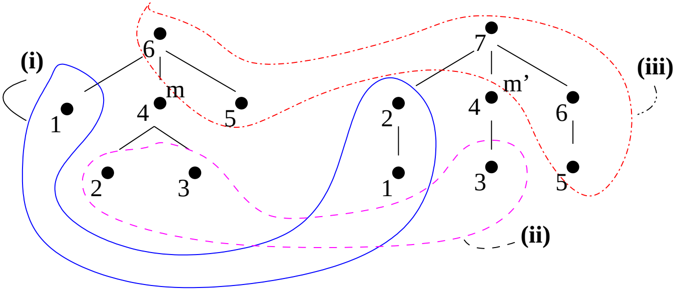

Boyer et al (2007) suggest the fectorisation

$$\sum_{e_1 \in E([\cdot_1],[\cdot_2(\cdot_1)])} [p(e_1)] \times \sum_{e_2 \in E([\cdot_2 \cdot_3],[\cdot_3])} [p(e_2)] \times p(m,m') \times \sum_{e_3 \in E([\cdot_6(\cdot_5)],[\cdot_7(\cdot_6(\cdot_5))])} [p(e_3)]$$

but we can show that this is not a sound factorisation

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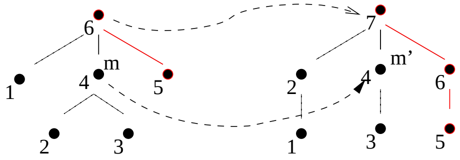

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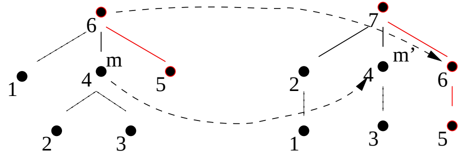

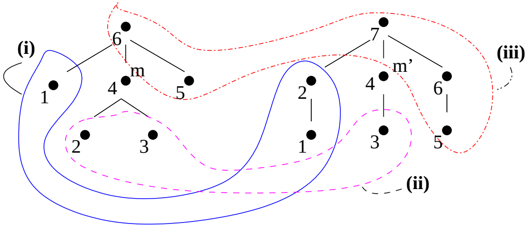

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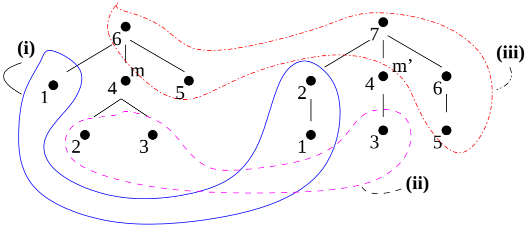

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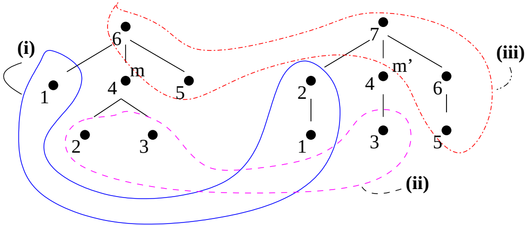
is the third term sums both things consistent and inconsistent with (m_4, m'_4)



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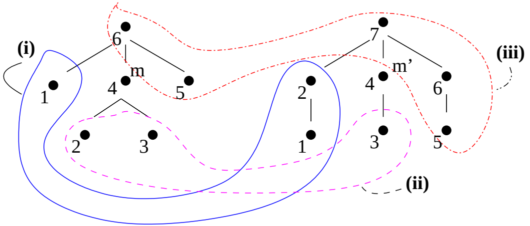

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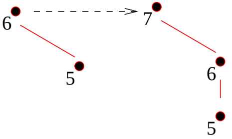
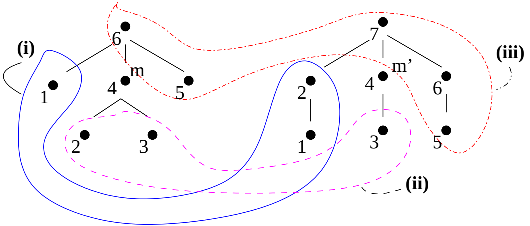


$+$

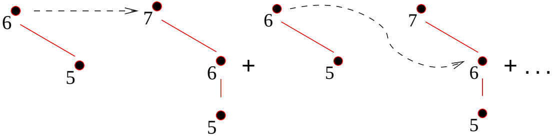


so the problem with the factorisation

$$\sum_{e_1\in E([\cdot_1],[\cdot_2(\cdot_1)])}[p(e_1)]\times\sum_{e_2\in E([\cdot_2\cdot_3],[\cdot_3])}[p(e_2)]\times p(m,m')\times\sum_{e_3\in E([\cdot_6(\cdot_5)],[\cdot_7(\cdot_6(\cdot_5))])}[p(e_3)]$$

is the third term sums both things consistent and inconsistent with (m_4, m'_4)

$\sum_{e_3\in E([\cdot_6(\cdot_5)],[\cdot_7(\cdot_6(\cdot_5))])}[p(e_3)] =$



For general trees, a feasible equivalent to the brute-force EM_A^{bf} remains an unsolved problem.

Outline

Let *the Viterbi EM algorithm* EM^V, be iterations of pair of steps

(Exp)$_V$ *generate a virtual corpus of scripts by treating each training pair* (S, T) *as standing for the* best *edit-script* σ, *which can relate* S *to* T, *weighting it by its conditional probability* $P(\sigma)/\Theta^A_s(S, T)$, *under current costs* $\mathcal{C}$

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(Max) *apply maximum likelihood estimation to the virtual corpus to derive a new probability table.*

Where $\mathcal{V}$ is the best-script, the virtual count or expectation $\gamma_{S,T}(op)$ contributed by S, T for the operation *op* is defined by

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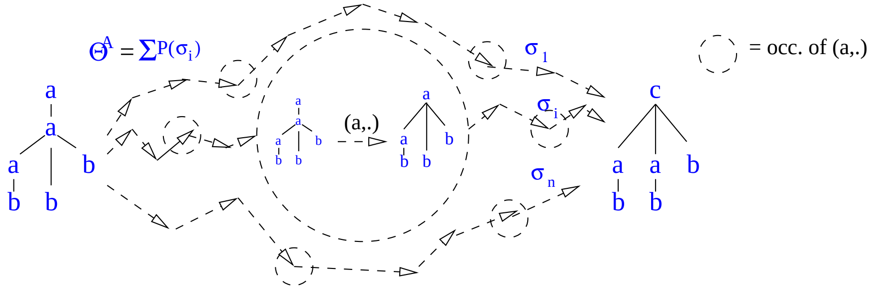
(Exp)$_V$ accumulates the $\gamma_{S,T}(op)$ for all *op*'s, for all (S, T)

Viterbi approximation EM^V (feasible)

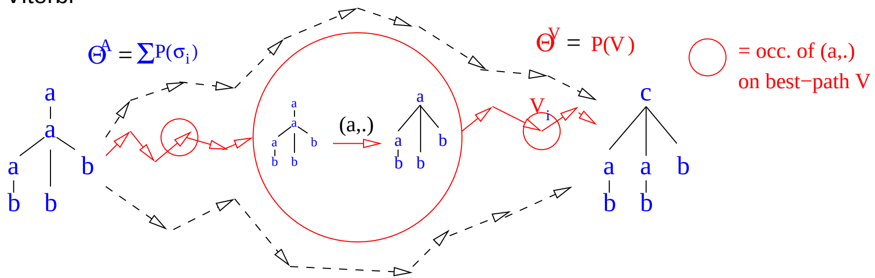
All paths



$$\gamma_{S,T}(op) = \sum_{\sigma:S\mapsto T}\left[\frac{P(\sigma)}{\Theta^A_s(S,T)} \times freq(op \in \sigma)\right]$$

Viterbi approximation EM^V (feasible)

Viterbi



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3. generate a corpus $\mathcal{TP}$ of tree pairs consistent with these edit scripts
4. apply learning algorithm to tree-pair corpus $\mathcal{TP}$ to learn parameters C' and compare to see if C' is close to original C.

Choosing a set of (target) parameters

- label alphabet $\Sigma = \{A, B, C, D, E\}$
- define subst. prob to be:
  max for letters one apart in ASCII code (eg A/B
  falling as you get further from this (eg $A/C < A/B$)

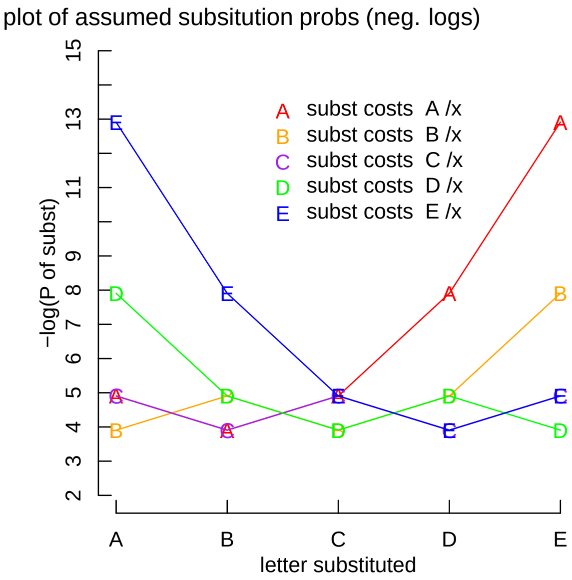
$$p(x, y) \;\; \alpha \;\; (|(|ASCII(x) - ASCII(y)| - 1)|)^2$$

- del and ins uniform, and such that ins+del ist just more than the worst swap

table (as neg. logs):

	λ	A	B	C	D	E
λ		6.907	6.907	6.907	6.907	6.907
A	6.907	4.907	3.907	4.907	7.907	12.91
B	6.907	3.907	4.907	3.907	4.907	7.907
C	6.907	4.907	3.907	4.907	3.907	4.907
D	6.907	7.907	4.907	3.907	4.907	3.907
E	6.907	12.91	7.907	4.907	3.907	4.907

The target parameters



Deriving a set of edit-scripts

generated 5k scripts in accordance with these parameters

it starts like this:

0 $[(A,D),(E,D),(D,\lambda),(C,D),(D,C),(A,B),(E,C),(D,C),(C,B),(C,D),(A,B),(C,A),($

1 $[(B,D)]$

2 $[(C,C),(D,B),(E,D),(B,D)]$

3 $[(D,B),(C,E),(A,A),(D,B),(C,B),(E,E),(C,D),(D,B),(\lambda,A),(E,C),(E,D)]$

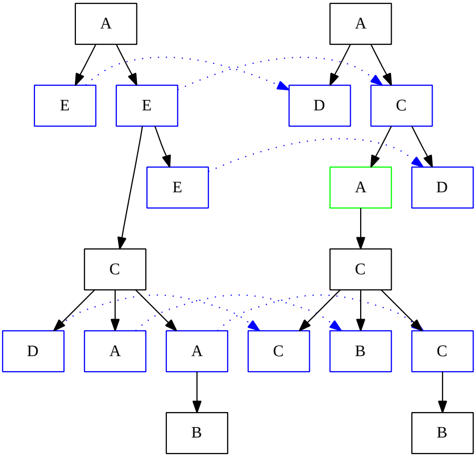
:

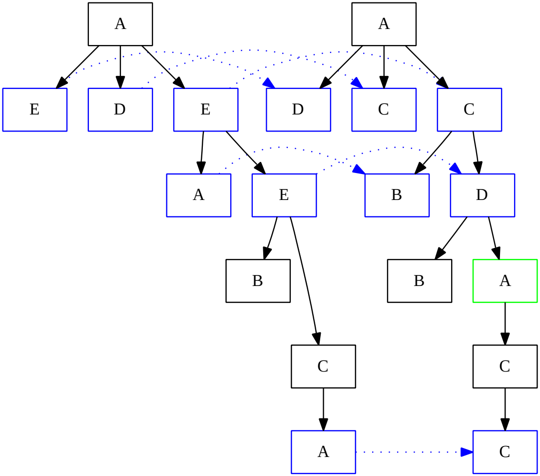
Generating tree pairs

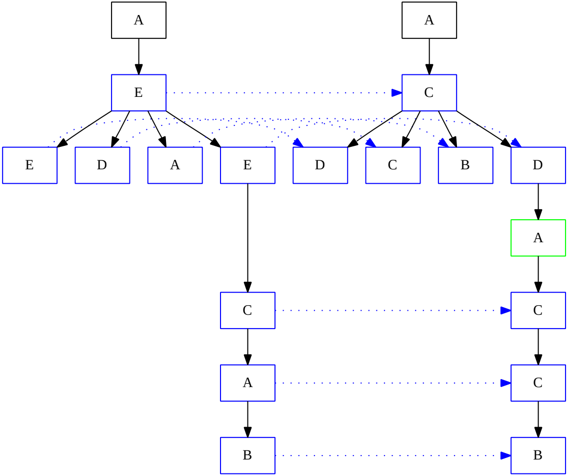
- from these scripts a corpus $\mathcal{TP}$ of consistent tree-pairs is generated
- for each script, a random 5 are chosen from all pairs consistent
- following pages for the script:

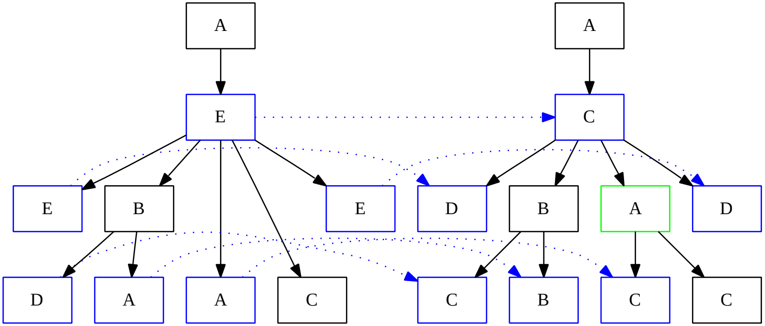
$$[(E,D)(D,C)(A,B)(B,B)(A,C)(C,C)([],A)(E,D)(E,C)(A,A)]$$

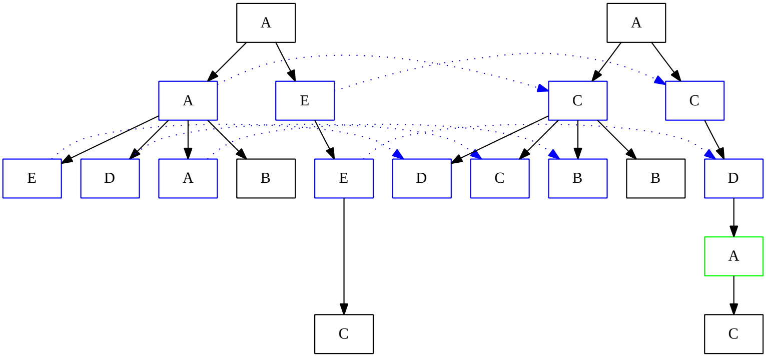
show the consistent tree pairs


A
A
E
E
D
C
E
A
D
C
C
D
A
A
C
B
C
B
B


A
A
E
D
E
D
C
C
A
E
B
D
B
B
A
C
C
A
C

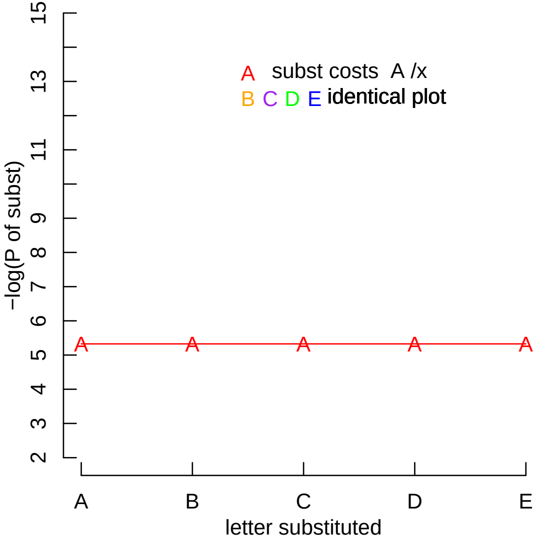

A
A
E
C
E
D
A
E
D
C
B
D
A
C
C
A
C
B
B


A
A
E
C
E
B
E
D
B
A
D
D
A
A
C
C
B
C
C


A
A
A
E
C
C
E
D
A
B
E
D
C
B
B
D
A
C
C

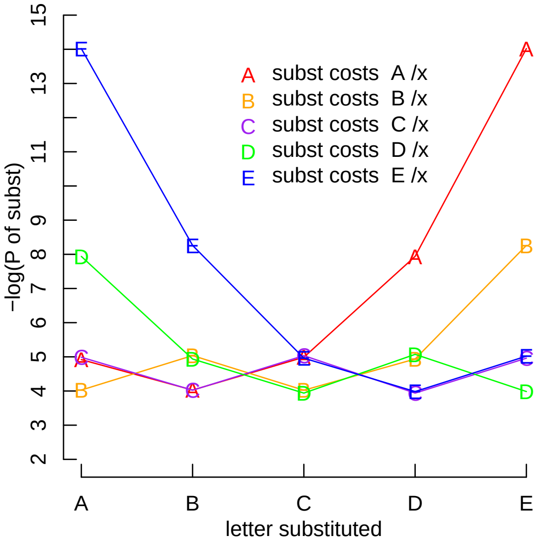
Applying training algorithm

Viterbi EM applied to corpus of tree pairs $\mathcal{TP}$
starting from initial uniform costs:



Applying training algorithm

Viterbi EM applied to corpus of tree pairs $\mathcal{TP}$ learns costs:



Outline

Adapting a k-NN classifier

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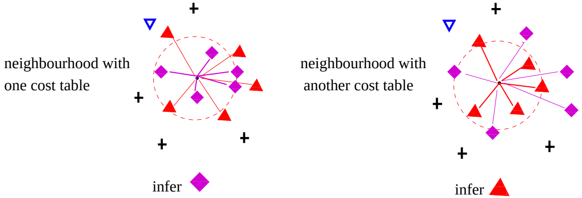

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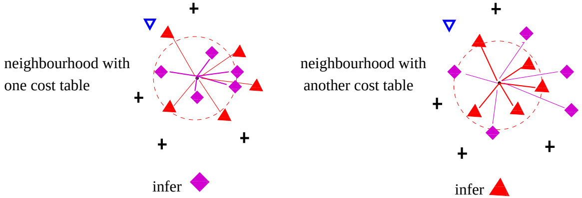

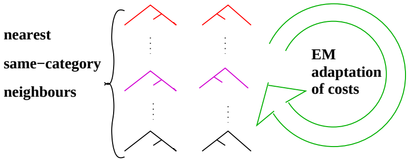

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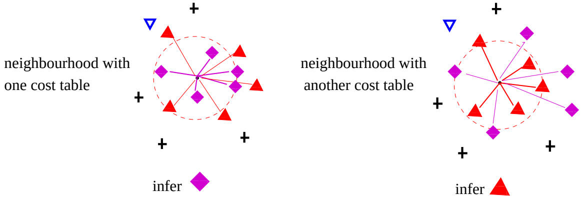

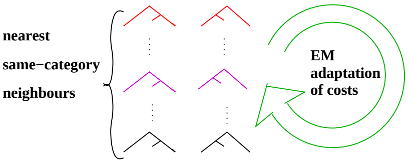

scripts between between *same-category neighbours should have distinctive probs* ⇒ perhaps can use *Expectation-Maximisation techniques to adapt edit-probs from a corpus of same-category nearest neighbours*

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2755 syntactically analysed and semantically categorised questions

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Cat	Example
NUM	When was London 's Docklands Light Railway constructed ? (SBARQ (WHADVP (WRB When))(SQ (VBD was)(NP (NP (NNP London)(POS 's))(NNPS Docklands) (JJ Light)(NN Railway))(VP (VBN constructed)))(. ?))
LOC	What country is the biggest producer of tungsten ? (SBARQ (WHNP (WDT What)(NN country))(SQ (VBZ is)(NP (NP (DT the)(JJS biggest)(NN producer)) (PP (IN of)(NP (NN tungsten)))))(. ?))
HUM	What is the name of the managing director of Apricot Computer ? (WHNP (WP What))(SQ (VBZ is)(NP (NP (DT the)(NN name))(PP (IN of)(NP (NP (DT the)(JJ managing)(NN director))

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Intuitiion: in scripts between between same-category neighbours *should* have distinctive probs eg. . $P(\textit{who}/\textit{when}) << P(\textit{state}/\textit{country})$.

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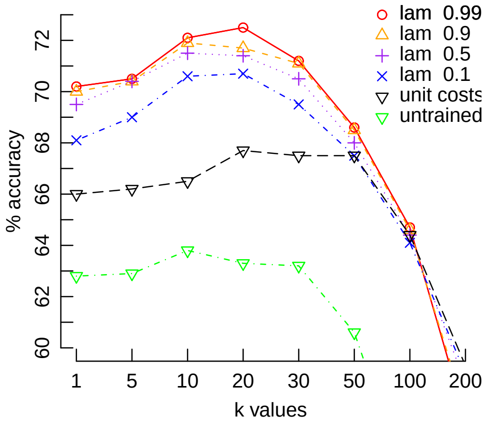
  stochastic tree distance with trained costs
  training by EM^V on same-category neighbours from the Example set

Experimental outcome (brief)



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- standard unit-costs
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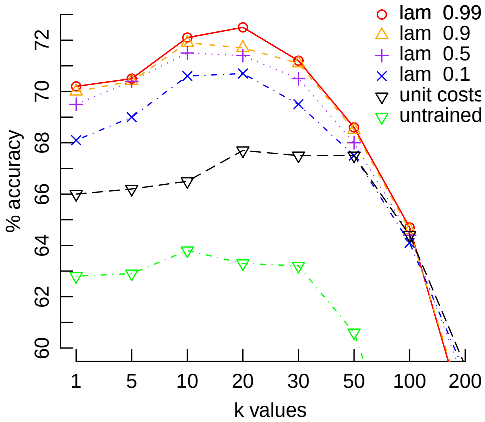

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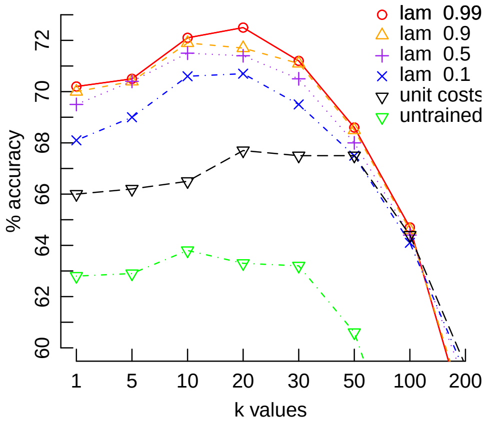
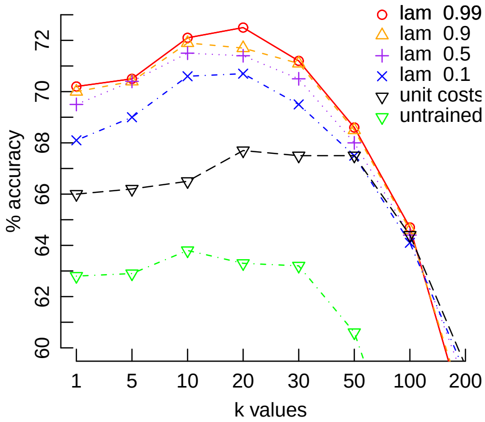


Experimental outcome (brief)

- standard unit-costs
  ▽, max. 67.7%
- initial stochastic costs
  ▽ max. 63.8%
  worse than unit costs
- best EM^V-adapted costs
  ○, max. 72.5%
  about 5% better than unit-costs
  (▽, max. 67.7%)



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Stochastic cost initialisation

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examples for d = 3, 10, 100, and 1000 are:

3	λ	a	b
λ	3.7	3.7	3.7
a	3.7	2.115	3.7
b	3.7	3.7	2.115

10	λ	a	b
λ	4.755	4.755	4.755
a	4.755	1.433	4.755
b	4.755	4.755	1.433

100	λ	a	b
λ	7.693	7.693	7.693
a	7.693	1.05	7.693
b	7.693	7.693	1.05

1000	λ	a	b
λ	10.97	10.97	10.97
a	10.97	1.005	10.97
b	10.97	10.97	1.005

NOTE: diagonal entries are not insignificant

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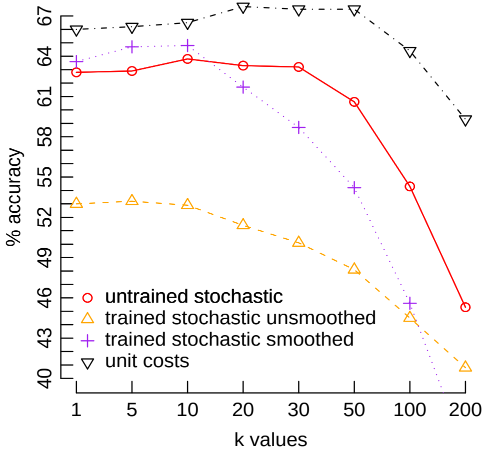
$$2^{-C^{\Delta}{}_{\lambda}[x][y]} = \lambda(2^{-C^{\Delta}[x][y]}) + (1-\lambda)(2^{-C^{\Delta}{}_{u}(d)[x][y]})$$

with $0 \leq \lambda \leq 1$

$\lambda = 1$ gives all the weight to the derived table

$\lambda = 0$ gives all the weight to the initial table

Experiment One



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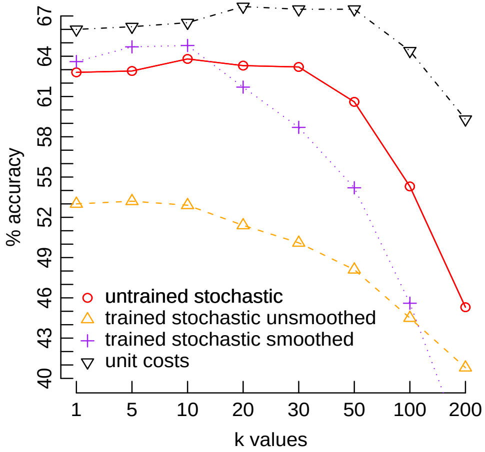
- unit-costs (▽, max. 67.7%) exceeds non-adapted $C^{\Delta}{}_{u}(3)$ costs (○, max. 63.8%)



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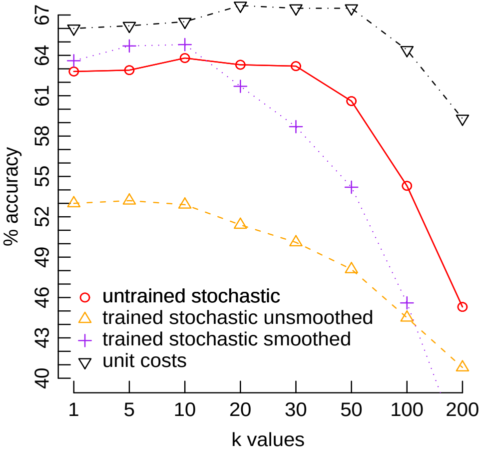

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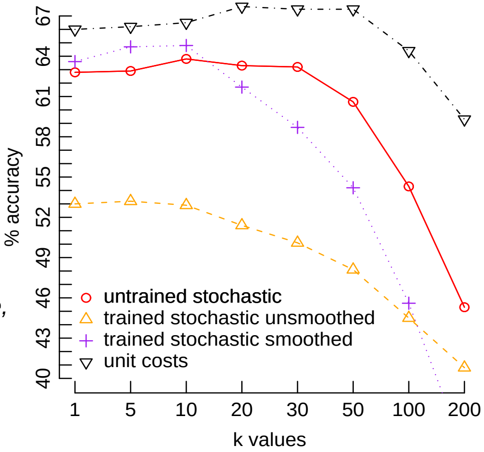
- EM^V-adapted costs on training set gives 95% accuracy: $\Rightarrow EM^V$ makes training pairs *too probable, and over-fits*.



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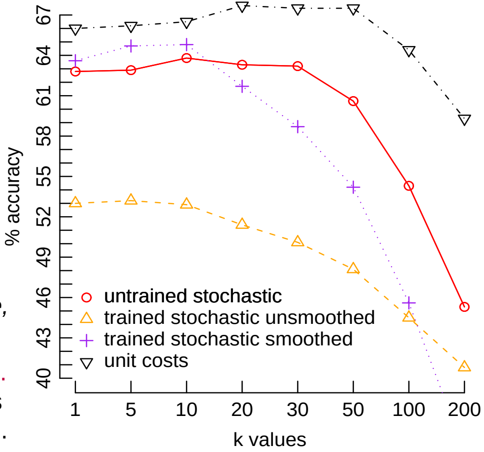
- EM^V-adapted costs on training set gives 95% accuracy: $\Rightarrow EM^V$ makes training pairs *too probable, and over-fits*.
- *smoothing* adapted costs (+,max. 64.8%) improves over initial costs (○) but is still below unit costs (▽).



Despite poor performace of the EM^V-adapted costs, some of the adapted costs seem intuitive. Here is a sample from top 1% of adapted swap costs, which are plausibly discounted relative to others:

8.50	?	.	12.31	The	the
8.93	NNP	NN	12.65	you	I
9.47	VBD	VBZ	13.60	can	do
9.51	NNS	NN	13.83	many	much
9.78	a	the	13.92	city	state
11.03	was	is	13.93	city	country
11.03	's	is			

Outline

- Recall: For the stochastic distance Δ_s^V cost-table entries represent probabilities via

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- suggests final step in which all the entries on the cost-table's diagonal are zeroed.

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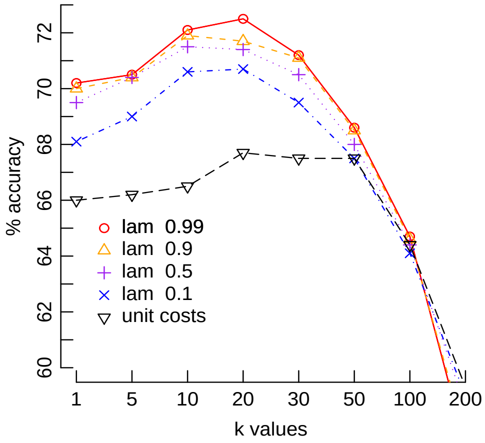
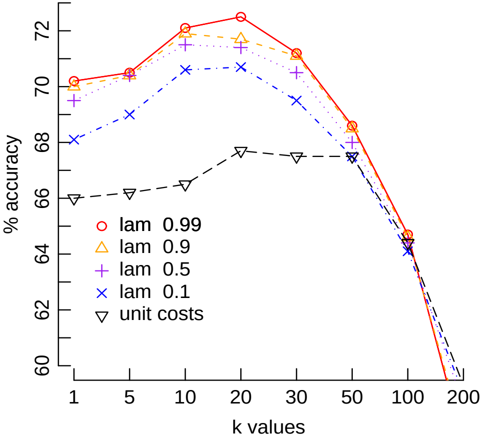
  $\Rightarrow$ a stochastically valid cost table cannot have zero costs on the diagonal
- perhaps this impedes good categorisation; note also the unit-cost setting, which is clearly 'uniform' in a sense, out-performs the 'uniform' stochastic initialisations
- suggests final step in which all the entries on the cost-table's diagonal are zeroed.
- Bilenko et al 2003 does essentially this in work on stochastic string distance

Experiment Two



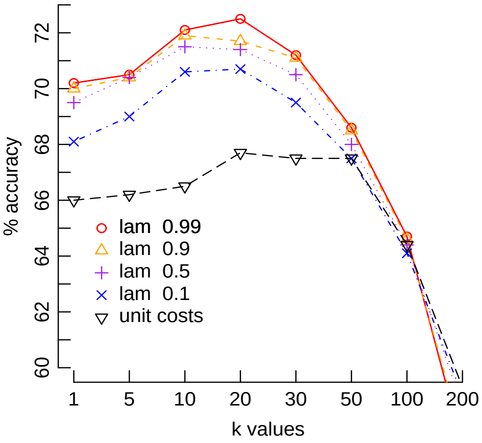
Experiment Two

- now with smoothing at varius levels of interpolation ($\lambda \in \{0.99, 0.9, 0.5, 0.1\}$) and with the diagonal zeroed, the *EM^V-adapted costs clearly out-perform* the unit-costs case ($\triangledown$).



Experiment Two

- now with smoothing at varius levels of interpolation ($\lambda \in \{0.99, 0.9, 0.5, 0.1\}$) and with the diagonal zeroed, the EM^V-adapted costs clearly out-perform the unit-costs case ($\bigtriangledown$).
- the best result being 72.5% ($k = 20$, $\lambda = 0.99$), as compared to 67.5% for unit-costs ($k = 20$)



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- experiments on further data-sets is required: one possibility is the NLP-related tasks of question-answering, where the need is to assess pairs of sentences for their likelihood to be a question-answer pairs. A training set of such pairs could also serve as potential input to the cost adaptation algorithm.

Literature

- *The paper this talks is mainly based on is* Emms (2011)
- *Background on string-distance and tree-distance*: Tai (1979) Zhang and Shasha (1989) Ristad and Yianilos (1998) Bilenko and Mooney (2003) Boyer et al. (2007)
- *Background on EM*: Prescher (2004)
- *Some work using similar approximation to all-paths EM*: Benedí and Sánchez (2005)
- *Question-bank*: Judge et al. (2006); Judge (2006a), Judge (2006b)
- *Some work using related models of stochastic tree-distance*: Takasu et al. (2007), Dalvi et al. (2009) Wang and Manning (2010)

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