μ -Challenge 1: Tricky Termination

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Consider the following recursively defined function:

$$\begin{array}{ccc} f & : & \mathbb{P} \ V \to \mathbb{N} \to V \\ f \ S \ i & \widehat{=} & \mathbf{if} \ \nu(i) \in S \ \mathbf{then} \ f \ S \ (i+1) \ \mathbf{else} \ \nu(i) \end{array}$$

where $\nu: \mathbb{N} \longrightarrow V$ is an injective function.

- 1. Prove that f terminates¹, if S is finite.
- 2. Function f also terminates if S is infinite, but satisfying some condition. What is that condition?
- 3. What is the function for, anyway?

 $^{^{1}\}mathrm{The}\ \mathrm{trick}$ is finding the well-founded relation.