Dowty's aspect hypothesis segmented

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A rough approximation from Rothstein 2004

activities $\lambda e.(DO(\varphi))(e)$

achievements $\lambda e.(\mathsf{BECOME}(\varphi))(e)$

 $\text{accomplishments} \quad \lambda e. \exists e'[(\mathsf{DO}(\varphi))(e') \land e = e' \sqcup_{\mathcal{S}} \mathsf{Cul}(e)]$

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For stative φ

$$\langle I, w \rangle \models \varphi \quad \text{iff} \quad (\forall t \in I) \ \langle \{t\}, w \rangle \models \varphi$$

contra φ for an event

 $\langle I, w \rangle \models \varphi$ iff I is the time of a φ -event in w

Idea. Bring out events by segmenting I to track change in stative φ 's

A segmentation of I is a sequence $I_1I_2\cdots I_n$ such that $I=\bigcup_{i=1}^n I_i$ and for $1\leq i < n,\ I_i \prec I_{i+1}$ — i.e. $(\forall t\in I_i)(\forall t'\in I_{i+1})\ t\prec t'$.



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A segmentation $I_1 \cdots I_n$ of I w-tracks φ if for all subintervals I' of I,

$$\langle I', w \rangle \models \varphi \quad \text{iff} \quad I' \subseteq \bigcup \{I_i \mid 1 \le i \le n \text{ and } \langle I_i, w \rangle \models \varphi\}.$$

A (φ, w, n) -alternation in I is a string $t_1 t_2 \cdots t_n \in I^n$ s.t. $t_i \prec t_{i+1}$ and $\langle \{t_i\}, w \rangle \models \varphi$ iff i is odd.

I is (φ, w) -alternation bounded (a.b.) if for some n > 0, no (φ, w, n) -alternation in *I* exists.

Fact. For stative φ ,

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$$I_1I_2 \models_w \boxed{\psi \ \varphi, \psi} \quad \text{iff} \quad \langle I_1, w \rangle \models \psi \text{ and } \langle I_2, w \rangle \models \varphi \wedge \psi$$
 $I_1 \cdots I_n \models_w \alpha_1 \cdots \alpha_m \quad \text{iff} \quad n = m \text{ and for } 1 \leq i \leq n \text{ and } \varphi \in \alpha_i,$
 $\langle I_i, w \rangle \models \varphi$

non-durative	durativ	ve	(length	\geq 3)	
achieve $\sim \varphi \mid \varphi$	accomplish	$\sim \varphi$	$\sim \varphi, \psi$	$\sim \varphi, \psi$	+ φ
semelfactive ψ	acti	ivity	$\psi \psi$	+	

 $\alpha_1 \cdots \alpha_n$ is *telic* if there is some φ in α_n such that the negation $\sim \varphi$ of φ appears in α_i for $1 \leq i < n$

Mary ran to post-office $\varphi = at(mary, post-office)$ not quantized (Krifka) . . . "arrow of time" (Landman & R 2012)

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- $(1) \quad \mathsf{iterate}(\boxed{\psi}) \ = \ \boxed{\psi\ \psi}^+$
- (2) $s\beta$; $\alpha s' := s(\beta \cup \alpha)s'$
- (3) $L; L' := \{s; s' \mid s \in L \{\epsilon\} \text{ and } s' \in L' \{\epsilon\}\}$
- (4) iterate(L) := (least $Z \supseteq L; L$) $Z; L \subseteq Z$
- (5) $\psi \psi ; \neg \varphi \varphi = \neg \varphi \neg \varphi, \psi \neg \varphi, \psi \varphi \mod inertial$

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With a state, unless something happens to change that state, then the state will continue ... With a dynamic situation, on the other hand, the situation will only continue if it is continually subject to a new input of energy.

$$\langle I, w \rangle \models \operatorname{Prev}(\varphi)$$
 iff $\langle I', w \rangle \models \varphi$ for some I' abutting I

$$\langle I, w \rangle \models \varphi \wedge \operatorname{Prev}(\sim \varphi)$$
 iff $I'I \models_{w} \overline{\sim \varphi} \varphi$ for some I'

$$\psi := \varphi \wedge \operatorname{Prev}(\sim \varphi)$$
, no segmentation w -satisfies any string in φ

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For $\psi \;:=\; arphi \wedge \mathsf{Prev}(\sim arphi)$, no segmentation w-satisfies any string in

$$iterate(\boxed{\psi}) = \boxed{\psi} \boxed{\psi}^+$$

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Incremental change and grain

$$\langle I, w \rangle \models d < \varphi \text{-deg} \quad \text{iff} \quad (\forall t \in I) \ d < \deg_{\varphi, w}^{D}(t)$$

$$\langle I, w \rangle \models \varphi \text{-deg} \leq d \quad \text{iff} \quad (\forall t \in I) \ \deg_{\varphi, w}^{D}(t) \leq d$$

$$\text{For } \psi := (\exists d \in D) \ (d < \varphi \text{-deg} \land \text{Prev}(\varphi \text{-deg} \leq d)),$$

$$I_0 I_1 \cdots I_n \models_w \boxed{\psi}^n \quad \text{iff} \quad (\exists d_1, \dots, d_n \in D)(\forall i \in [1, n])$$

$$(\forall t \in I_{i-1})(\forall t' \in I_i)$$

$$\deg_{\varphi, w}^{D}(t) \leq d_i < \deg_{\varphi, w}^{D}(t')$$

$$\begin{array}{c|c} \varphi\text{-deg} \leq d_1 & d_1 < \varphi\text{-deg}, \ \varphi\text{-deg} \leq d_2 \end{array} \cdots \\ \cdots & d_{n-1} < \varphi\text{-deg}, \ \varphi\text{-deg} \leq d_n & d_n < \varphi\text{-deg} \end{array}$$

Grain is fixed by the set of propositions we can box.

$$\langle I, w \rangle \models d < \varphi \text{-deg} \quad \text{iff} \quad (\forall t \in I) \ d < \deg^D_{\varphi, w}(t)$$

$$\langle I, w \rangle \models \varphi \text{-deg} \leq d \quad \text{iff} \quad (\forall t \in I) \ \deg^D_{\varphi, w}(t) \leq d$$

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$$|I_0 I_1 \cdots I_n \models_w \boxed{\psi}^n \quad \text{iff} \quad (\exists d_1, \dots, d_n \in D)(\forall i \in [1, n])$$

$$(\forall t \in I_{i-1})(\forall t' \in I_i)$$

$$\deg^D_{\varphi, w}(t) \leq d_i < \deg^D_{\varphi, w}(t')$$

$$egin{aligned} arphi_{-} \mathrm{deg} & \leq d_1 \ d_1 < arphi_{-} \mathrm{deg}, \ arphi_{-} \mathrm{deg} \leq d_2 \ \cdots \ & \cdots \ d_{n-1} < arphi_{-} \mathrm{deg}, \ arphi_{-} \mathrm{deg} \leq d_n \ d_n < arphi_{-} \mathrm{deg} \end{aligned}$$

$$\langle I,w
angle \models d < arphi$$
-deg iff $(\forall t\in I)\ d < \deg^D_{arphi,w}(t)$
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For $\psi:=(\exists d\in D)\ (d-deg \wedge Prev $(arphi$ -deg $\leq d)$),
 $I_0I_1\cdots I_n\models_w \boxed{\psi}^n$ iff $(\exists d_1,\ldots,d_n\in D)(\forall i\in [1,n])$
 $(\forall t\in I_{i-1})(\forall t'\in I_i)$
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$$arphi$$
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The projections
$$bc_A(s) := bc(\rho_A(s))$$

 $o_{\mathcal{A}}$ "see only \mathcal{A} "

 $b\!c$ "no time without change" : compress $lpha^+$ to lpha

For infinite Φ , let $Fin(\Phi)$ be the set of finite subsets of Φ .

A Φ -system is a function $f: Fin(\Phi) \to (2^{\Phi})^*$ such that

$$(\forall B \in Fin(\Phi))(\forall A \subseteq B) \quad f(A) = bc_A(f(B)).$$

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The projections
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days in a year → months in a year

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bc "no time without change" : compress a^+ to a^-

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$\mathbb{IL}(\Phi)$ branches

For
$$f, f' \in \mathbb{IL}(\Phi)$$
, $f \prec_{\Phi} f'$ iff $f \neq f'$ and $(\forall A \in Fin(\Phi))$ $f(A)$ is a prefix of $f'(A)$ where

s is a prefix of s' iff
$$(\exists s'') s' = ss''$$

Fact. \prec_{Φ} is transitive and left linear: for all $f \in \mathbb{L}(\Phi)$, and $f_1, f_2 \prec_{\Phi} f$,

$$f_1 \prec_{\Phi} f_2$$
 or $f_2 \prec_{\Phi} f_1$ or $f_1 = f_2$

Moreover, no Φ -system is \prec_{Φ} -maximal.

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An interval world pair $\langle I, w \rangle$ is Φ -approximable if every $\varphi \in \Phi$ is homogeneous and alternation-bounded in $\langle I, w \rangle$.

- Fact. Every Φ -approximable $\langle I, w \rangle$ is representable in $\mathbb{H}(\Phi)$ by a unique system $\{f(A)\}_{A \in Fin(\Phi)}$ of approximations of $\langle I, w \rangle$ as $f(A) \in (2^A)^*$ at granularity A.
- DAH_s: At granularity A, events within $\langle I, w \rangle$ are representable as substrings of f(A)
 - an *E*-event occurs in $\langle I, w \rangle$ iff $(\exists s \in L_E)$ $s \sqsubseteq f(A)$ relational intension: $s' \supseteq_E s$ iff $s \in L_E$ and $s \sqsubseteq s'$

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