Predications, fast & slow

Tim.Fernando@tcd.ie

Commonsense-2017, London

DANIEL KAHNEMAN, Thinking, Fast & Slow, 2011

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	subject	predicate	Description Logic
Tweety flies	individual	concept	flies(Tweety)
Birds fly	concept	concept	$\mathit{bird} \sqsubseteq \mathit{flies}$

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WILLIAM WOODS, Meaning & Links, 2007

extensional vs intensional subsumption

predication	subsumption
fast	intensional
slow	extensional

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fast	intensional
slow	extensional

1. path \sim string

≈ model of *Monadic Second-Order Logic* (MSO)

MSO-sentence \approx regular language (BÜCHI, ELGOT & TRAKHTENBROT)

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Inheritance & inertia as: No change without reason (Principle of Sufficient Reason, Leibniz)

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2. extensions approximated at bounded but refinable granularity

What You See Is All There Is (WYSIATI, KAHNEMAN)

- satisfaction condition for institution (Goguen & Burstall 1992)

1 Intensions vs extensions

2 Paths & MSO

Granularity & institutions

	subject	predicate	predication
Descr Logic	individual	concept	∈ (ABox)
FCA context	object	attribute	HAS

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FCA: Given a set D of objects and a set A of attributes,

INTENT(
$$D$$
) := { $a \mid (\forall d \in D) \ d \text{ HAS } a$ }
EXTENT(A) := { $d \mid (\forall a \in A) \ d \text{ HAS } a$ }

a concept is a pair
$$(D, A)$$
 s.t. $A = INTENT(D)$ & $D = EXTENT(A)$

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- a concept is a pair (D, A) s.t. A = INTENT(D) & D = EXTENT(A)
 - equivalently, A = INTENT(EXTENT(A))
 - for concepts A and A',

$$\mathsf{EXTENT}(A) \subseteq \mathsf{EXTENT}(A') \iff A' \subseteq A$$

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- for concepts A and A',

$$\mathsf{EXTENT}(A) \subseteq \mathsf{EXTENT}(A') \iff A' \subseteq A$$

- for each object d, INTENT $(\{d\})$ is a concept

Inheritance

$$d\sqsubseteq d'\iff \mathrm{INTENT}(\{d'\})\subseteq \mathrm{INTENT}(\{d\})$$

$$\frac{d'\;\mathrm{has}\;a\quad d\sqsubseteq d'}{d\;\mathrm{has}\;a} \qquad \mathrm{INTENT}(D)\;:=\;\{a\,|\,(\forall d\in D)\;d\;\mathrm{has}\;a\}$$

Inheritance qualified

$$d\sqsubseteq d'\iff \mathrm{INTENT}(\{d'\})\subseteq \mathrm{INTENT}(\{d\})$$

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- exceptions: birds fly but not penguins ...

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- exceptions: birds fly but not penguins \dots

$$\frac{d' \text{ HAS } a \qquad d \text{ IS } d' \qquad \text{not}(d \text{ HAS } \overline{a})}{d \text{ HAS } a}$$

d HAS $\overline{a} \neq \operatorname{not}(d$ HAS a) every penguin is flightless $\neq \operatorname{not}(\text{every penguin flies})$

Inheritance qualified

$$d \sqsubseteq d' \iff \text{INTENT}(\{d'\}) \subseteq \text{INTENT}(\{d\})$$

$$\frac{d' \text{ has } a \quad d \sqsubseteq d'}{d \text{ has } a} \qquad \text{INTENT}(D) := \{a \mid (\forall d \in D) \text{ } d \text{ has } a\}$$

- exceptions: birds fly but not penguins ...

$$\frac{d' \text{ HAS } a \qquad d \text{ IS } d' \qquad \text{not}(d \text{ HAS } \overline{a})}{d \text{ HAS } a} \text{ in}(a)$$

 $d \; {
m HAS} \; \overline{a} \;
eq {
m not} (d \; {
m HAS} \; a)$ every penguin is flightless $eq {
m not} ({
m every \; penguin \; flies})$

category mistake: widespread birds but *Tweety ...

G. CARLSON: individual/kind/stage-level predication

Tweety flies
Birds are widespread
Tweety was thirsty

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- generics are less about instances than about

rules & regulations, "causal forces behind instances" (1995)

CARLSON & STEEDMAN causes

- G. CARLSON: individual/kind/stage-level predication Birds are widespread Tweety was thirsty
 - generics are less about instances than about rules & regulations, "causal forces behind instances" (1995)
- M. STEEDMAN 2005: temporality is about "causality & goal-directed action"

die(Tweety) contra inertial alive(Tweety)

CARLSON & STEEDMAN causes

- G. CARLSON: individual/kind/stage-level predication Birds are widespread Tweety was thirsty
 - generics are less about instances than about rules & regulations, "causal forces behind instances" (1995)

Intensions from instances/extensions

From INTENT(
$$\{d\}$$
) = A with

$$A = INTENT(EXTENT(A))$$

$$A \sqsubseteq A' \iff \text{EXTENT}(A) \subseteq \text{EXTENT}(A')$$

Intensions from instances/extensions to strings/causes

From INTENT $(\{d\}) = A$ with

$$A = \text{intent}(\text{extent}(A))$$

 $A \sqsubseteq A' \iff \text{extent}(A) \subseteq \text{extent}(A')$

to strings

$$A_1 \cdots A_n$$
 with $A_n = A$ $A_{n-1} pprox ext{INTENT}(\{d'\})$ for $d'Sd$ \cdots

S from top/past for inferences such as

$$\frac{a \in A_i}{a \in A_{i+1}} \ 1 \le i < n$$

for d'Sd saying $d ext{ IS } d'$.

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Attributes in strings

FCA	string $A_1 \cdots A_n$	
d HAS a	$a \in A_i$	
object <i>d</i>	position <i>i</i>	
attribute <i>a</i>		

Attributes in strings as predicates

FCA	string $A_1 \cdots A_n$	MSO
d HAS a	$a \in A_i$	$i \in \llbracket P_a rbracket$
object <i>d</i>	position <i>i</i>	$i \in \{1, \ldots, n\}$
attribute a		unary predicate P_a

$$[\![P_a]\!] = \{i \in \{1,\ldots,n\} \mid a \in A_i\}$$

Attributes in strings as predicates

FCA	string $A_1 \cdots A_n$	MSO _.
d HAS a	$a \in A_i$	i ∈ [[P _a]]
object <i>d</i>	position <i>i</i>	$i \in \{1, \ldots, n\}$
attribute $a \in \mathcal{A}$	$A_i\subseteq \mathcal{A}$	unary predicate P_a

$$[\![P_a]\!] = \{i \in \{1, \dots, n\} \mid a \in A_i\}$$
$$A_i = \{a \in A \mid i \in [\![P_a]\!]\}$$

Attributes in strings as predicates

FCA	string $A_1 \cdots A_n$	$MSO_\mathcal{A}$
d HAS a	$a \in A_i$	$i \in \llbracket P_a rbracket$
object <i>d</i>	position <i>i</i>	$i \in \{1, \ldots, n\}$
attribute $a\in\mathcal{A}$	$A_i\subseteq \mathcal{A}$	unary predicate P_a

 MSO_A -model = string over the alphabet 2^A

$$A_1 \cdots A_n \models \exists x (P_a x \land \forall y \neg y S x) \iff a \in A_1$$

 $\mathsf{MSO}\text{-}\mathsf{sentence} = \mathsf{regular} \ \mathsf{language} \quad \big(B \ddot{\mathtt{U}} \mathtt{CHI} \ \ldots \big)$

$$\frac{a \in A_i \qquad \overline{a} \not\in A_{i+1}}{a \in A_{i+1}}$$

$$(P_a y \wedge \neg P_{\overline{a}} x \wedge y S x) \supset P_a x$$

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$$P_a x \quad \mapsto \quad \exists X (Xx \land \mathsf{path}_a(X))$$

$$\mathsf{path}_a(X) \ := \ \underbrace{\forall x (Xx \supset \exists y (y S x \land Xy) \lor P_a x)}_{X \ \mathsf{backs} \ \mathsf{up}^S \ \mathsf{until} \ a} \quad \bigwedge \underbrace{\neg \exists x (Xx \land P_{\overline{a}} x)}_{X \ \mathsf{avoids} \ \overline{a}}$$

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$$\frac{a \in A_i \quad o(a) \notin A_i}{a \in A_{i+1}} \qquad (P_a y \land \neg P_{o(a)} y \land y S x) \supset P_a x$$

$$\frac{a \in A_{i} \quad \overline{a} \not\in A_{i+1}}{a \in A_{i+1}} \qquad (P_{a}y \land \neg P_{\overline{a}}x \land ySx) \supset P_{a}x$$

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$$P_a x \mapsto \exists X (Xx \land \mathsf{path}_a^o(X))$$

$$\mathsf{path}^o_a(X) \ := \ \forall x (Xx \supset \exists y (ySx \land Xy \land \neg P_{o(a)}y) \lor P_ax)$$

Finite state transducers down S

Fix a finite set *In* of inheritable/inertial attributes.

$$\frac{a \in A_i \qquad \overline{a} \not\in A_{i+1}}{a \in A_{i+1}}$$

 $\mathsf{state} = \mathsf{subset} \ q \ \mathsf{of} \ \mathit{In} \ \mathsf{in} \ \mathsf{previous} \ \mathsf{position} \ (\mathsf{initially} \ \emptyset)$

$$q \xrightarrow{A:A'} q'$$
 where $A' := A \cup \{a \in q \mid \overline{a} \notin A\}$
 $q' := A' \cap In$

Finite state transducers down S

Fix a finite set In of inheritable/inertial attributes.

$$\frac{a \in A_i \quad \overline{a} \not\in A_{i+1}}{a \in A_{i+1}}$$

state = subset q of In in previous position (initially \emptyset)

$$q \xrightarrow{A:A'} q'$$
 where $A' := A \cup \{a \in q \mid \overline{a} \notin A\}$
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$$\frac{a \in A_i \qquad o(a) \notin A_i}{a \in A_{i+1}}$$

$$q \xrightarrow{A:A'} q'$$
 where $A' := A \cup q$
 $q' := \{a \in A' \cap In \mid o(a) \not\in A\}$

A causal ontology

Trade GALOIS connection

$$D \subseteq \text{EXTENT}(A) \iff A \subseteq \text{INTENT}(D)$$

for an ontology based on S-change

$$(P_a y \wedge y S x) \supset (P_a x \vee y R_a x)$$
 $y R_a x := \begin{cases} P_{\overline{a}} x & \text{for kinds} \\ P_{o(a)} y & \text{for time} \end{cases}$

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Principle of Sufficient Reason (Leibniz) $\begin{cases} \text{differentia } \overline{a} \\ \text{force } o(a) \end{cases}$

bias for $P_a x \approx$ a domain minimisation assumption

A causal ontology based on attributes

Trade GALOIS connection

$$D \subseteq \text{EXTENT}(A) \iff A \subseteq \text{INTENT}(D)$$

for an ontology based on S-change $(a \in In)$

$$(P_a y \wedge y S x) \supset (P_a x \vee y R_a x)$$
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Principle of Sufficient Reason (Leibniz) $\left\{ \begin{array}{l} \text{differentia } \overline{a} \in \mathit{In} \\ \text{force } o(a) \not \in \mathit{In} \end{array} \right.$

bias for $P_a x \approx$ a domain minimisation assumption

$$\frac{\mathsf{individual}}{\mathsf{kind}} \approx \frac{\mathsf{instant}}{\mathsf{interval}} \approx \frac{\mathsf{stative}}{\mathsf{eventive}} \approx \frac{\mathsf{persistent}}{\mathsf{altering}} \approx \frac{\forall \; (\mathsf{homogeneous})}{\exists \; (\mathsf{ontological})}$$

1 Intensions vs extensions

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Granularity & institutions

Reducts

Given a set
$$\mathcal{A}$$
 of attributes and $A \subseteq \mathcal{A}$, A -reduct of $\langle \{1,\ldots,n\}, S_n, \llbracket P_a \rrbracket_{a \in \mathcal{A}} \rangle$ is $\langle \{1,\ldots,n\}, S_n, \llbracket P_a \rrbracket_{a \in \mathcal{A}} \rangle$ $\rho_A(A_1 \cdots A_n) := (A_1 \cap A) \cdots (A_n \cap A)$ "see only A "
$$\rho_{\{a,\overline{a}\}}(\boxed{a,b} \boxed{a} \boxed{\overline{a},c}) = \boxed{a} \boxed{a} \boxed{\overline{a}}$$

Reducts & compression

Given a set \mathcal{A} of attributes and $A \subseteq \mathcal{A}$, A-reduct of $\langle \{1, \ldots, n\}, S_n, \llbracket P_a \rrbracket_{a \in \mathcal{A}} \rangle$ is $\langle \{1, \ldots, n\}, S_n, \llbracket P_a \rrbracket_{a \in \mathcal{A}} \rangle$ $\rho_A(A_1 \cdots A_n) := (A_1 \cap A) \cdots (A_n \cap A)$ "see only A"

$$\rho_{\{a,\overline{a}\}}(\overline{a,b}|\overline{a},\overline{a},c)) = \overline{a}|\overline{a}|\overline{a}$$

$$bc(\rho_{\{a,\overline{a}\}}(\overline{a,b}|\overline{a},\overline{a},c)) = \overline{a}|\overline{a}|$$

Compress $A_1 \cdots A_n$ to eliminate stutters $A_i A_{i+1}$ with $A_i = A_{i+1}$

$$\mathfrak{k}(A_1 \cdots A_n) := \begin{cases}
A_1 & \text{if } n = 1 \\
\mathfrak{k}(A_2 \cdots A_n) & \text{else if } A_1 = A_2 \\
A_1 \, \mathfrak{k}(A_2 \cdots A_n) & \text{otherwise}
\end{cases}$$

Reducts & compression

Given a set \mathcal{A} of attributes and $A \subseteq \mathcal{A}$, A-reduct of $\langle \{1,\ldots,n\},S_n,\llbracket P_a \rrbracket_{a \in \mathcal{A}} \rangle$ is $\langle \{1,\ldots,n\},S_n,\llbracket P_a \rrbracket_{a \in \mathcal{A}} \rangle$ $\rho_A(A_1\cdots A_n) := (A_1\cap A)\cdots(A_n\cap A)$ "see only A" $\rho_{\{a,\overline{a}\}}(\boxed{a,b} \boxed{a} \boxed{a},c) = \boxed{a} \boxed{a}$ $\mathfrak{b}(\rho_{\{a,\overline{a}\}}(\boxed{a,b} \boxed{a} \boxed{a},c)) = \boxed{a} \boxed{a}$

Compress $A_1 \cdots A_n$ to eliminate stutters $A_i A_{i+1}$ with $A_i = A_{i+1}$

$$\mathfrak{w}(A_1 \cdots A_n) := \begin{cases}
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\mathfrak{w}(A_2 \cdots A_n) & \text{else if } A_1 = A_2 \\
A_1 \mathfrak{w}(A_2 \cdots A_n) & \text{otherwise}
\end{cases}$$

Base ontology on granularity

$$bc_A(s) := bc(\rho_A(s))$$

An $\mathsf{MSO}_{\mathcal{A}}$ -formula φ has finite $\mathit{voc}(\varphi) \subseteq \mathcal{A}$ with all attributes in φ

$$A_1 \cdots A_n \models \varphi \iff \rho_{voc(\varphi)}(A_1 \cdots A_n) \models \varphi$$

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satisfaction condition (GOGUEN & BURSTALL) for an

institution
$$\begin{cases} \text{ signature } A = \text{ finite subset of } \mathcal{A} \\ A\text{-model} = \text{string over the alphabet } 2^{A} \\ A\text{-sentence} = \text{MSO}_{A}\text{-sentence} \end{cases}$$

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What You See Is All There Is (WYSIATI, KAHNEMAN)

For finite-state transducer T for inheritance,

$$bc_{In}(T(A_1\cdots A_n)) = bc(T(bc_{In}(A_1\cdots A_n)))$$

and similarly for inertia.

An MSO_A-formula φ has finite $voc(\varphi) \subseteq A$ with all attributes in φ

$$A_1 \cdots A_n \models \varphi \iff \rho_{voc(\varphi)}(A_1 \cdots A_n) \models \varphi$$

satisfaction condition (GOGUEN & BURSTALL) for an

What You See Is All There Is (WYSIATI, KAHNEMAN)

For finite-state transducer T for inheritance,

$$\omega_{ln}(T(A_1\cdots A_n)) = \omega(T(\omega_{ln}(A_1\cdots A_n)))$$

and similarly for inertia.

Multiple A-models — bound search by reducing A but additional constraints may expand A and change institution

- 1. strings/causes in place of instances/extensions
 - strings as MSO-models
 - expect finite automata to be fast

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 - strings as MSO-models
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- 2. top-down, contra bottom-up
 - given xSy $\begin{cases} x \text{ is more general than } y \\ x \text{ is before } y \end{cases}$ draw inference from x to y
 - avoid fixing an extension

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 $\begin{cases} x \text{ is more general than } y \\ x \text{ is before } y \end{cases}$
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 - strings as MSO-models
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- given
$$xSy$$
 $\begin{cases} x \text{ is more general than } y \\ x \text{ is before } y \end{cases}$

draw inference from x to y

- avoid fixing an extension
- 3. from known unknowns to unknown unknowns

 A-models change of signature, institution