

Booth's Algorithm

Partial Product

Multiplier	Partial Product
X_{i+1}, X_i, X_{i-1}	Rational
0 0 0	no 1 => add zero
0 0 1	end of string
0 1 0	string of length 1
0 1 1	end of string
1 0 0	start of string
1 0 1	end/start of string
1 1 0	start of string
1 1 1	continuation of string

Partial Product	weighted with
+0	2^i
+Y	
+Y	
+2Y	
-2Y	
-y	$[-2y + Y = -Y]$
-y	
-0	

Shift and ADD

Demonstration of Booth [Init]

Let $X = 111101 = -3$
 $Y = 000111 = +7$

Reg. S.A.Q = 13 bit shift register
 $Q_{-1} = \text{overlap}$

+Y = 000111
 -Y = 111001

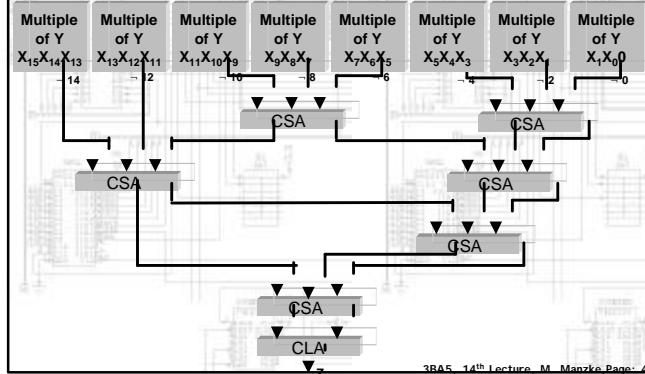
Shift and ADD

Demonstration of Booth

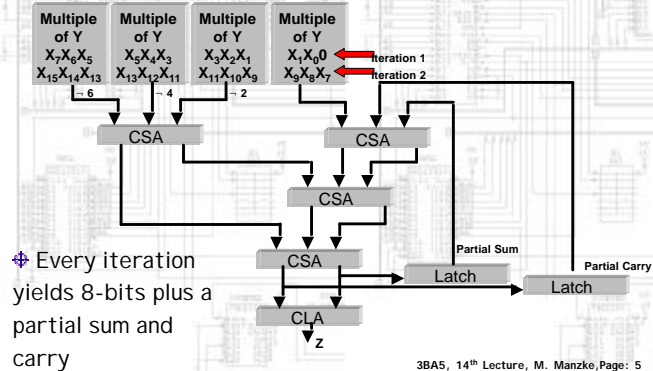
S	A	Q	Q_{-1}	Remark
0	000 000	111 101.0		$S.A, Q_{-1} \rightarrow 0, Q \leftarrow X$
0	000 111	---	---	$A \leftarrow A + Y$
0	000 001	111 111.0		$2 * \text{ashr}(S.A.Q)$
1	111 010	---	---	$A \leftarrow A - Y$
1	111 110	101 111.1		$2 * \text{ashr}(S.A.Q)$
-	---	---	---	$A \leftarrow A - 0$
1	111 111	101 011.1		$2 * \text{ashr}(S.A.Q)$

Parallel Generation of Partial Products and their Sumation

$Z = X * Y, X$ 16bit



Multi Pass Booth-Wallace



Floating-point Numbers

$$X = \pm m \cdot b^e$$



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b = 2
e = exponent - bias
bias = 2j-k-1 - 1
if (m != 0) then m = 1.mantissa
if (s = 0) then X > 0
if (s = 1) then X < 0

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Evaluating Biased Exponents



Exponent = e + bias
 = e + 127, e(8 bit)

<u>Exponent</u>	<u>Decimal</u>	<u>Binary</u>
-126	-126 + 127 = 1	00000001
-001	-001 + 127 = 126	01111110
000	000 + 127 = 127	01111111
001	001 + 127 = 128	10000000
126	126 + 127 = 253	11111101
127	127 + 127 = 254	11111110

Normalisation

- ✦ In order to preserve maximum precision non-zero members have their exponent adjusted so that their MSB is always 1.
- ✦ Hence it does not need to be stored
- ✦ Is called the hidden bit

- ⊕ In 1982 twenty different floating-point formats were in use
 - ⊕ Causing serious portability problems
- ⊕ In 1985 this standard, derived from a 1979 paper by Kahan, Stone and Coonen, was adopted and remains.



Precision	I	j	k	e(bits)	m(bits)	bias
Single	31	30	22	8	24	$127=2^7-1$
Double	63	62	51	11	53	$1023=2^{10}-1$

$$\text{Single } (-1)^S \cdot (1.m) \cdot 2^{e-127}$$

$$\text{Double } (-1)^S \cdot (1.m) \cdot 2^{e-1023}$$